Before-After Studies

Part 2

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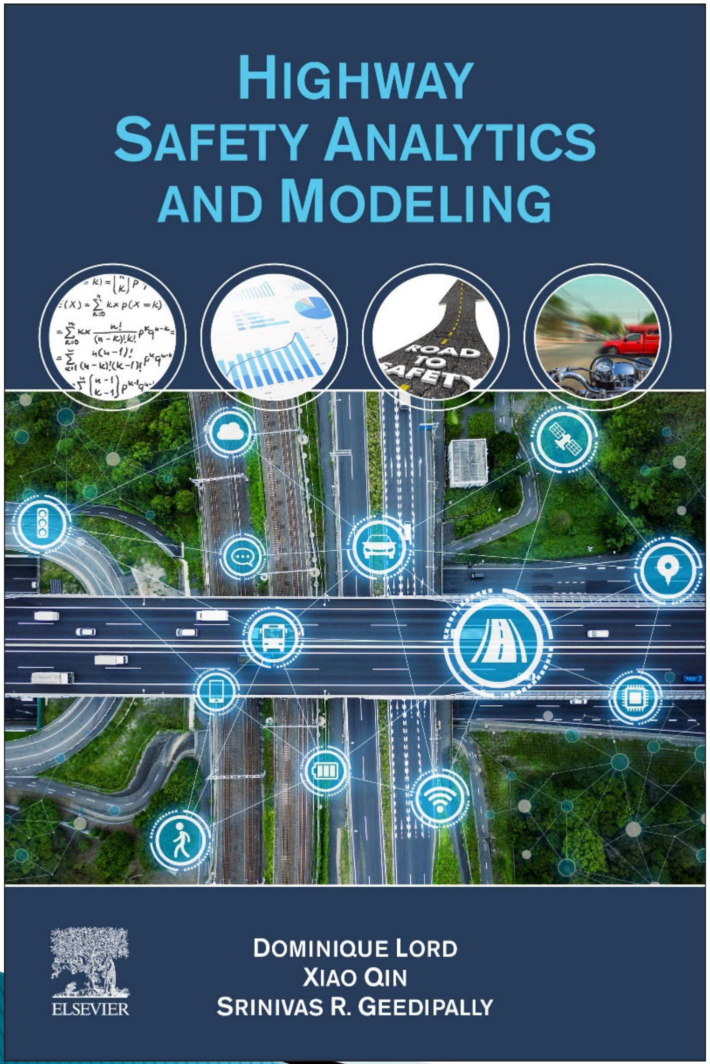
Textbook



The material presented in this series of lectures are taken from this textbook and other sources based on lectures given by the authors.

The textbook is available on Amazon and the Elsevier website below among other places.

https://www.elsevier.com/books/highway-safety-analytics-and-modeling/lord/978-0-12-816818-9

Quick Recap

- Important Issues
  - RTM and Selection Bias
- Prediction and Estimation
- Comparison of Prediction and Estimation
  - Difference and Ratio (Index)
- Naïve Method and Method with Comparison/Reference Group

Regression-to-the-mean

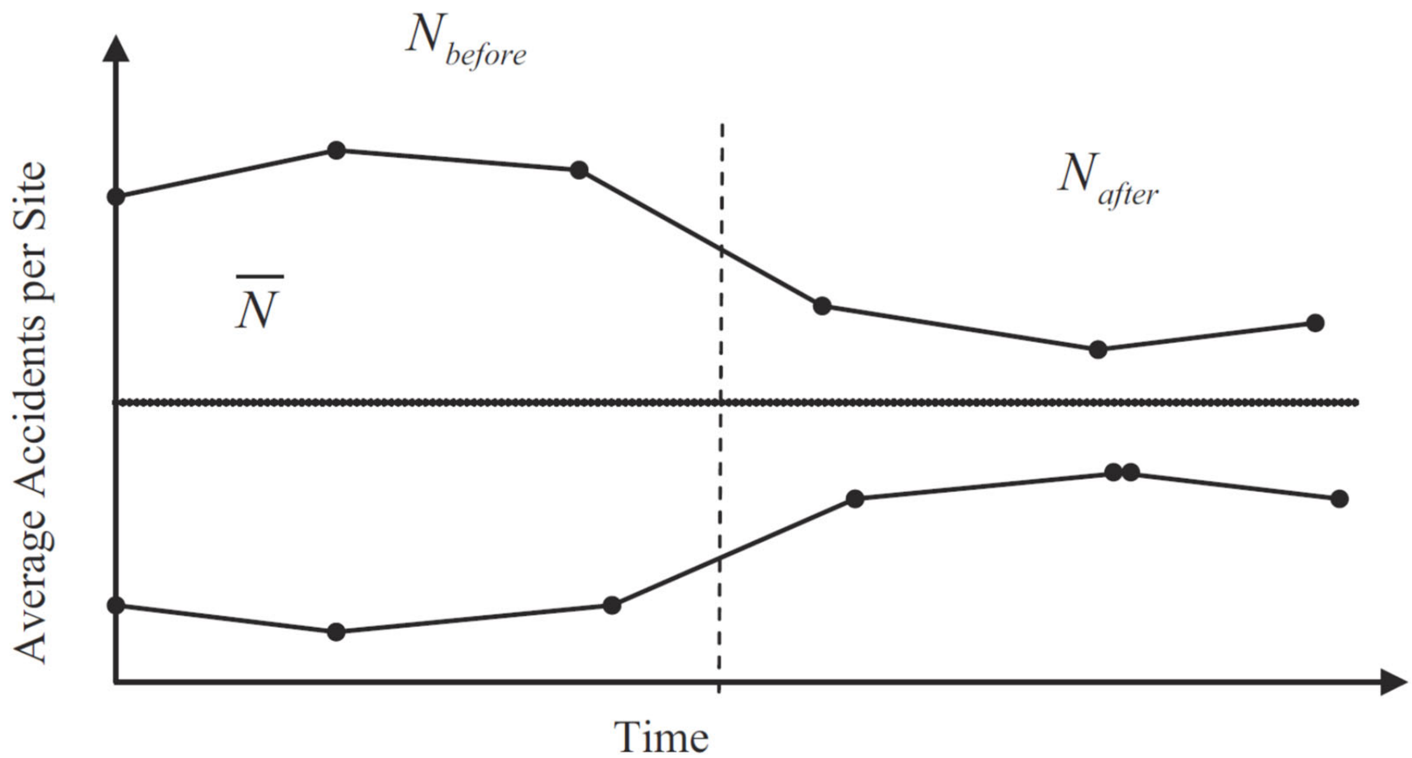


FIGURE 7.1 Representation of the regression-to-the-mean (Lord and Kuo, 2012).

Site Selection Bias

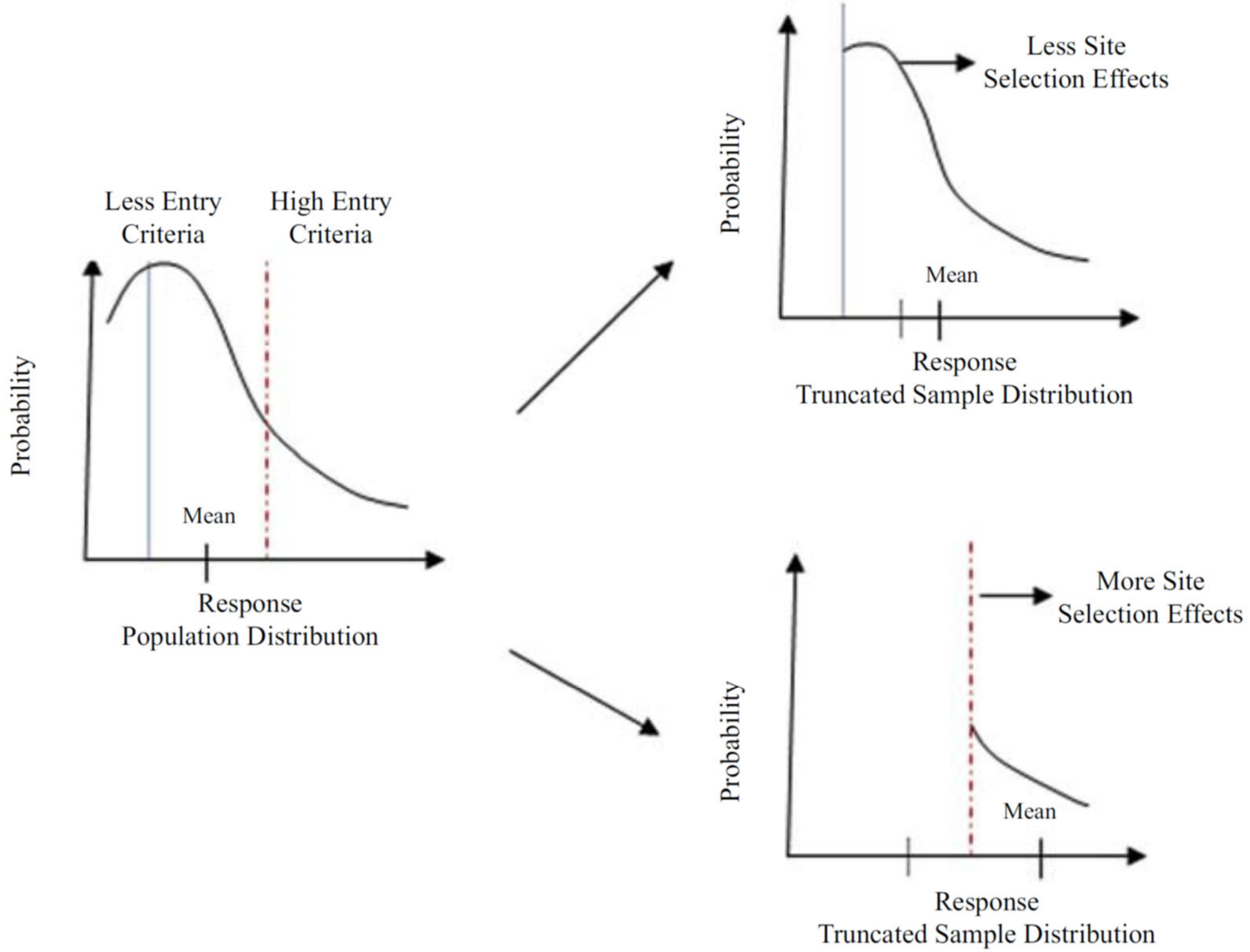


FIGURE 7.3 The population distribution for complete and truncated samples (Lord and Kuo, 2012).

Before-After Studies

There are many variants of Before-After studies. They can be regrouped under two tasks:

1. Predict what would have been the safety of an entity in the "after" period, had the treatment not been applied, and
2. Estimate what the safety of the treated entity in the after period was.

An entity is a general term used to designate a road section, intersection, ramp, driver, etc.

The analysis can be divided into four basic steps.

Before-After Studies

First, we need to define the notation that will be used for performing the two tasks at hand.

Let:

π be the expected number of target crashes of a specific entity in an after period would have been had it not been treated; π is what must be predicted.

λ be the expected number of target crashes of a specific entity in an after period; λ is what must be estimated.

Before-After Studies

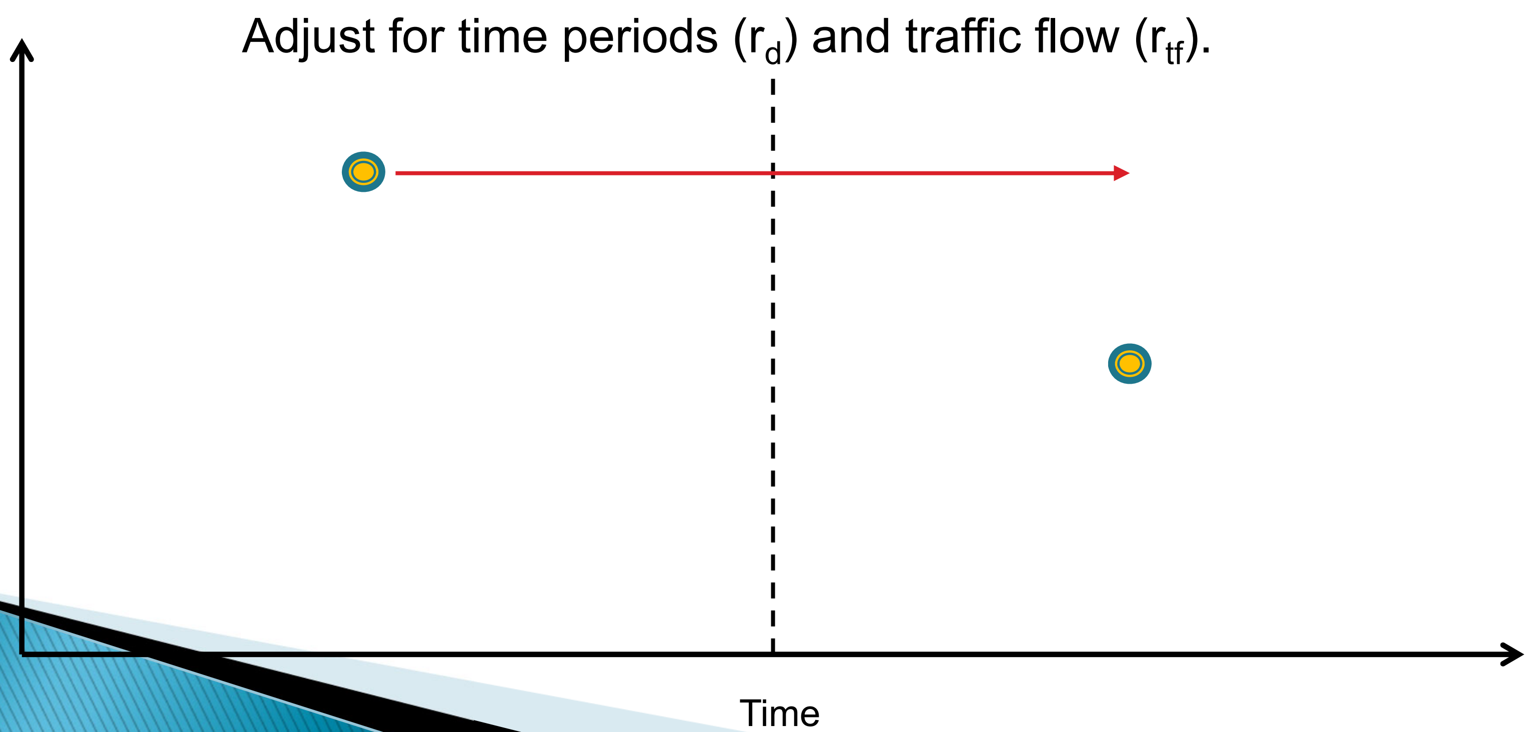
The effect of a treatment is judge by comparing π and λ . The two comparisons we are usually interested are the following:

$\delta = \pi - \lambda$ the reduction in the after period of the expected number of target crashes (by kind and severity).

$\theta = \lambda / \pi$ the ratio of what was the treatment to what it would have been without the treatment; this is defined as the index of effectiveness.

Naïve/Simple Before-After Studies

In its simplest form, an observational before-after study consists of comparing the counts occurring in the before period to its count in the after period. The term naïve stands for the fact that counts in the before period are used as predictor of the expected crashes occurring in the after period.



Before-After Studies with Comparison Group

Let us define the following notations:

$$r_c = \nu / \mu$$

The ratio of the expected crash counts for the comparison group

$$r_t = \pi / \kappa$$

The ratio of the expected crash counts for the treatment group

The hope is that $r_t = r_c$ $\quad \therefore \pi = r_c \kappa = r_t \kappa$

$$\omega = r_c / r_t$$

Odd's ratio

Time periods need to be the same for both the comparison and treatment groups

Empirical Bayes Method

- Premise: the safety of a site is estimated using two sources of information:
  - 1) information obtained from sites that have the same characteristics (reference population)
  - 2) information obtained from the actual site where the EB method is being applied
- Reference population
  - Method of moments (covered in PIARC RSM 2003 – very rarely used now)
  - **Statistical model**

Empirical Bayes Method

Formulation:

$$\mu_{EB} = \gamma\mu + (1-\gamma)y$$

where

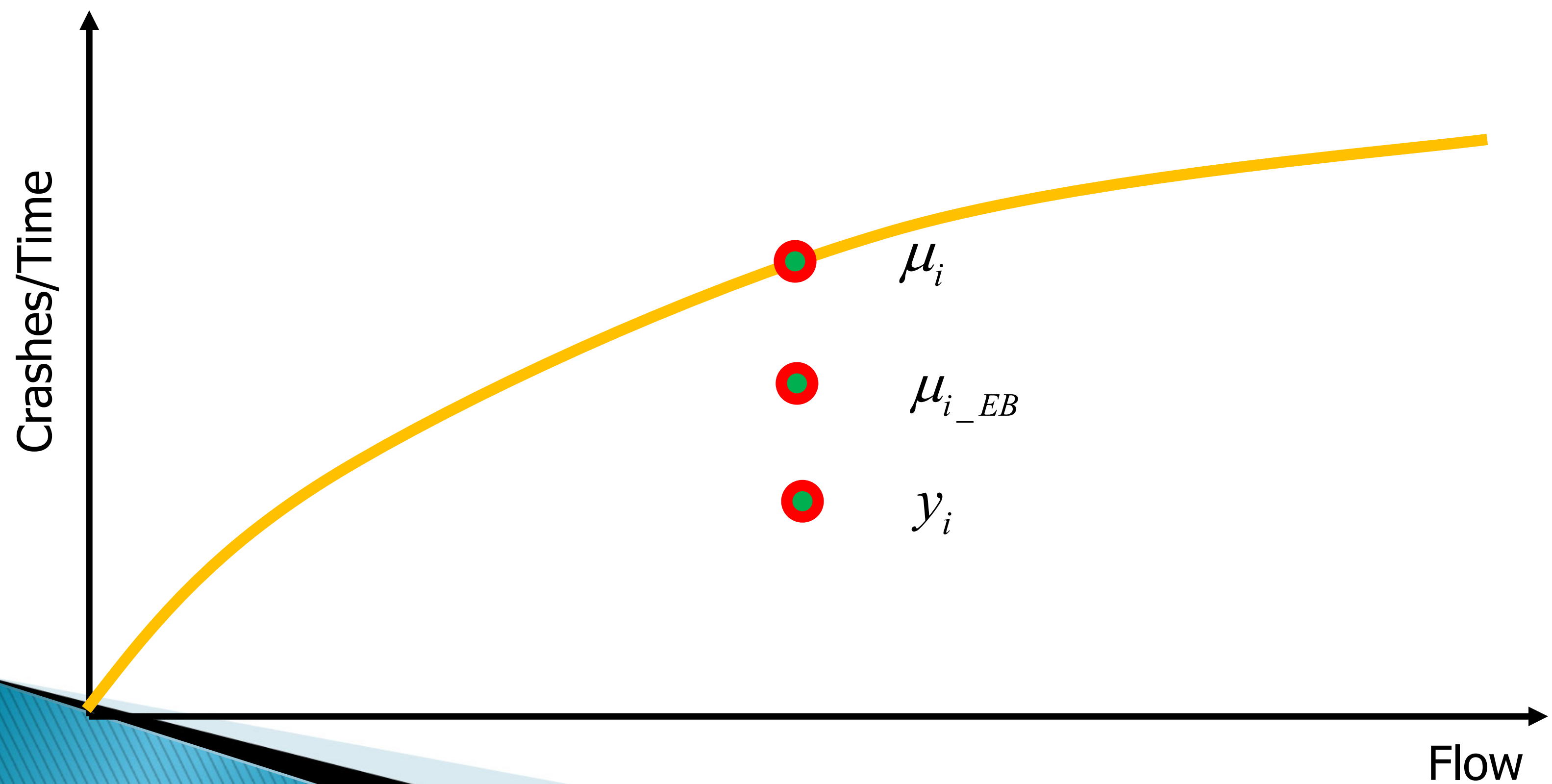
Mean

$$\gamma = \frac{1}{1+\dfrac{\mu}{\phi}}$$

Dispersion parameter

Empirical Bayes Method

In the context of safety estimation, the EB method is assumed to more accurately estimate the long-term mean of a given site. Recall that the simplified assumption states that crashes for a given site/observations follow a Poisson distribution (over time) where the mean is gamma distributed (or other distributions).



Empirical Bayes Method

Estimating μ using a statistical model

$$\mu = \exp\left(\mathbf{x}'\boldsymbol{\beta}\right)$$

For the EB method, the most used model remains the NB model, but recently other models have been proposed such as the Sichel, PIG, and NB-L among others.

Last month, two papers have proposed a different approach for estimating the EB estimate: 1) simulation-based EB (random parameters) and 2) non-parametric EB method.

Empirical Bayes Method

Formulation of the variance (based on NB):

$$Var\{\mu\} = \frac{\mu^2}{\phi}$$

$$Var\{\mu_{EB}\} = \left(1-\gamma\right)\mu_{EB}$$

The EB Variance

Empirical Bayes Method

STEP 1: Develop statistical models.

Using data from the control group, develop one or several statistical models.

From the model(s), estimate the dispersion parameter ϕ .

$$\mu = \exp\left(\mathbf{x}'\boldsymbol{\beta}\right)$$

Empirical Bayes Method

STEP 2: Estimate μ_{EB} and $Var\{\mu\}_{EB}$ for the before period.

$$\mu_{EB} = \frac{\left(\phi + y_b\right)}{\left(\phi/\mu + t_b\right)}$$

μ_{EB} = expected annual number of crashes for the before period

y_b = crash count during the period "t" years (labeled as t_b)

$$Var\{\mu_{EB}\} = \left(1-\gamma\right)\mu_{EB}$$

Empirical Bayes Method

STEP 3: Estimate r_{tf}.

$$r_{tf} = f(A) \big/ f(B)$$

$$f(A) = \mu_a = \exp(\mathbf{x\beta})$$

For each site, use the characteristics for the after period

$$f(B) = \mu_b = \exp(\mathbf{x\beta})$$

For each site, use the characteristics for the before period

Empirical Bayes Method

STEP 4: Estimate the number of collision for the after period.

$$\pi = r_{tf} \times t_a \times \mu_{EB}$$

t_a = the number of years for the after period

Empirical Bayes Method

STEP 5: Estimate λ . (same as before)

STEP 6: Estimate $Var(\lambda)$ and $Var(\pi)$.

$$Var(\lambda) = \lambda$$

$$Var(\pi) = \frac{\mu_{EB} \times \left(r_{tf} \times t_a\right)^2}{\left(\phi/\mu + t_b\right)}$$

Empirical Bayes Method

STEP 7: Estimate δ and θ using the output from STEP 4, STEP 5 and STEP 6.

$$\delta = \pi - \lambda$$

$$\theta = \frac{\lambda}{\pi\left[1 + Var\{\pi\} / \pi^2\right]}$$

Empirical Bayes Method

STEP 8: Estimate $Var\{\delta\}$ and $Var\{\theta\}$.

$$Var\{\delta\} = Var\{\pi\} + Var\{\lambda\}$$

$$Var\{\theta\} \approx \frac{\theta^2 \left[\left(Var\{\lambda\} / \lambda^2 \right) + \left(Var\{\pi\} / \pi^2 \right) \right]}{\left[1 + Var\{\pi\} / \pi^2 \right]^2}$$

Empirical Bayes Method

Example Application

Example taken from "**Observational Before-After Study of the Safety Effect of U.S. Roundabout Conversions Using the Empirical Bayes Method**" by Persaud et al. (2001) in Transportation Research Record 1751, pp. 1-8.

The objective was to estimate the changes in motor vehicle crashes following conversion of 23 intersections from stop sign and traffic signal control to modern roundabouts.

Empirical Bayes Method

Sites where a roundabout was built.

TABLE 1 Details of the Sample of Roundabout Conversions

Jurisdiction	Year Opened	Control Before[a]	Single or Multilane	AADT Before	AADT After	Months Before	Months After	Crash Count Before All	Crash Count Before Injury	Crash Count After All	Crash Count After Injury
Anne Arundel County, MD	1995	1	Single	15,345	17,220	56	38	34	9	14	2
Avon, CO	1997	2	Multilane	18,942	30,418	22	19	12	0	3	0
Avon, CO	1997	2	Multilane	13,272	26,691	22	19	11	0	17	1
Avon, CO	1997	6	Multilane	22,030	31,525	22	19	44	4	44	1
Avon, CO	1997	6	Multilane	18,475	27,525	22	19	25	2	13	0
Avon, CO	1997	6	Multilane	18,795	31,476	22	19	48	4	18	0
Bradenton Beach, FL	1992	1	Single	17,000	17,000	36	63	5	0	1	0
Carroll County, MD	1996	1	Single	12,627	15,990	56	28	30	8	4	1
Cecil County, MD	1995	1	Single	7,654	9,293	56	40	20	12	10	1
Fort Walton Beach, FL	1994	2	Single	15,153	17,825	21	24	14	2	4	0
Gainesville, FL	1993	6	Single	5,322	5,322	48	60	4	1	11	3
Gorham, ME	1997	1	Single	11,934	12,205	40	15	20	2	4	0
Hilton Head, SC	1996	1	Single	13,300	16,900	36	46	48	15	9	0
Howard County, MD	1993	1	Single	7,650	8,500	56	68	40	10	14	1
Manchester, VT	1997	1	Single	13,972	15,500	66	31	2	0	1	1
Manhattan, KS	1997	1	Single	4,600	4,600	36	26	9	4	0	0
Montpelier, VT	1995	2	Single	12,627	11,010	29	40	3	1	1	1
Vail, CO	1995	1	Multilane	15,300	17,000	36	47	16	n/a	14	2
Vail, CO	1995	4	Multilane	27,000	30,000	36	47	42	n/a	61	0
Vail, CO	1997	4	Multilane	18,000	20,000	36	21	18	n/a	8	0
Vail, CO	1997	4	Multilane	15,300	17,000	36	21	23	n/a	15	0
Washington County, MD	1996	1	Single	7,185	9,840	56	35	18	6	2	0
West Boca Raton, FL	1994	1	Single	13,469	13,469	31	49	4	1	7	0

[a]1 = four-legged, one street stopped; 2 = three-legged, one street stopped; 4 = other unsignalized; 6 = signal

Empirical Bayes Method

Sites where a roundabout was built.

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Avon, CO	1997	6	Multilane	18,795	31,476	22	19	48	4	18	0
Bradenton Beach, FL	1992	1	Single	17,000	17,000	36	63	5	0	1	0
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Cecil County, MD	1995	1	Single	7,654	9,293	56	40	20	12	10	1
Fort Walton Beach, FL	1994	2	Single	15,153	17,825	21	24	14	2	4	0
Gainesville, FL	1993	6	Single	5,322	5,322	48	60	4	1	11	3
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Howard County, MD	1993	1	Single	7,650	8,500	56	68	40	10	14	1
Manchester, VT	1997	1	Single	13,972	15,500	66	31	2	0	1	1
Manhattan, KS	1997	1	Single	4,600	4,600	36	26	9	4	0	0
Montpelier, VT	1995	2	Single	12,627	11,010	29	40	3	1	1	1
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[a]1 = four-legged, one street stopped; 2 = three-legged, one street stopped; 4 = other unsignalized; 6 = signal

Empirical Bayes Method

Sites used as reference group for calibrating NB regression models.

TABLE 2 Details of the Data Set Used to Calibrate Regression Models

Intersection Class	Jurisdiction	Number of Intersections	Years of Data	Range of Minor Road AADT	Range of Major Road AADT	Total Crashes	Injury Crashes
Four-legged	Maryland	18	10	365–3,133	8,625–52,144	597	177
	Florida	9	6	1,064–3,487	15,017–39,558	228	79
	Toronto	59	6	384–8,487	5,755–52,598	1,317	357
	All	86		365–8,487	5,755–52,598	2,142	613
Three-legged	Maryland	3	10	858–1,992	21,294–40,535	177	64
	Florida	3	6	722–2,006	16,012–25,905	64	27
	Toronto	117	6	105–7,771	9,101–51,725	1,690	472
	All	123		105–7,771	9,101–51,725	1,931	563

Empirical Bayes Method

STEP 1: Develop statistical models.

Recalibrated original regression model (functional form) by Bonneson and McCoy:

$$E(m) = 0.692\left(\frac{T_m}{1{,}000}\right)^{0.256}\left(\frac{T_c}{1{,}000}\right)^{0.831}$$

$$u = 0.000379 \times (\text{major road AADT})^{0.256} \times (\text{minor road AADT})^{0.831} \quad \phi = 4.0$$

The model above is for rural 4-legged 2-stop controlled intersections.

Other models for signalized and three-legged intersections were calibrated for the project (see paper and previous slide).

Empirical Bayes Method

STEP 2: Estimate μ_{EB} and $Var\{\mu\}_{EB}$ for the before period.

TABLE 6 Data for Example Conversion

	Before Conversion	After Conversion
Months (years) of crash data	56 (4.67)	38 (3.17)
Count of total crashes	34	14
Major approaches AADT	10,654	11,956
Minor approaches AADT	4,691	5,264

$$P\,(crashes/year) = 0.000379 \times (major\ road\ AADT)^{0.256} \times (minor\ road\ AADT)^{0.831}$$
$$= 0.000379 \times (10{,}654)^{0.256} \times (4{,}691)^{0.831} = 4.58.$$

$$\phi = 4.0$$

$$P = \mu$$

$$m_b = (k + x_b) / (k/P + y_b),$$

$$m_b = \mu_{EB}$$

$$m_b = (4.0 + 34) / [(4/4.58) + 4.67] = 6.860.$$

$$x_b = t_b$$

$$\gamma = \frac{1}{1+\frac{\mu}{\phi}} = \frac{1}{1+\frac{6.86}{4.0}} = 0.37 \longrightarrow Var\{\mu\}_{EB} = (1-0.37)\times 6.86 = 4.33$$

Empirical Bayes Method

STEP 3: Estimate r_{tf} .

$$P\,(crashes/year) = 0.000379 \times (major\ road\ AADT)^{0.256} \times (minor\ road\ AADT)^{0.831}$$
$$= 0.000379 \times (10{,}654)^{0.256} \times (4{,}691)^{0.831} = 4.58.$$

Before

$$crashes/year = 0.000379 \times (11{,}956)^{0.256} \times (5{,}264)^{0.831} = 5.19.$$

After

$$R = 5.19/4.58 = 1.133,$$

Empirical Bayes Method

STEP 4: Estimate the number of collision for the after period.

$$m_a = R \times m_b = 1.133 \times 6.860 = 7.772 \text{ crashes/year.}$$

$$B = 7.772 \times 3.17 = 24.63.$$

Number of years after

$$Var(B) = (m_b) \times (R \times y_a)^2 / [(k/P) + y_b]$$
$$= 6.860 \times (1.133 \times 3.17)^2 / (0.873 + 4.67) = 15.96$$

Empirical Bayes Method

STEP 5: Estimate λ . (same as before)

TABLE 7 Empirical Bayes Estimates for Five Maryland Conversions

After Period Count (A)	Empirical Bayes Estimate (B)	$Var(B)$
14	36.71	30.63
14	24.63	15.96
2	14.38	9.40
10	14.33	8.55
4	15.16	6.76
Sum = λ = 44	Sum = π =105.21	Sum = 71.30

Empirical Bayes Method

STEP 5: Estimate λ . (same as before)

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Sum = λ = 44	Sum = π =105.21	Sum = 71.30

Empirical Bayes Method

STEP 6: Estimate $Var(\lambda)$ and $Var(\pi)$.

$$Var(\lambda) = \lambda \longrightarrow Var(\lambda) = 14$$

$$Var(\pi) = \frac{\mu_{EB} \times \left(r_{tf} \times t_a\right)^2}{\left(\phi / \hat{\mu} + t_b\right)}$$

$$Var(B) = (m_b) \times (R \times y_a)^2 / [(k/P) + y_b]$$
$$= 6.860 \times (1.133 \times 3.17)^2 / (0.873 + 4.67) = 15.96$$

Empirical Bayes Method

STEP 7: Estimate δ and θ using the output from STEP 4, STEP 5 and STEP 6.

TABLE 7 Empirical Bayes Estimates for Five Maryland Conversions

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14	36.71	30.63
14	24.63	15.96
2	14.38	9.40
10	14.33	8.55
4	15.16	6.76
Sum = λ = 44	Sum = π =105.21	Sum = 71.30

$$\delta = 105.21 - 44 = 61.21.$$

$$\theta = (44/105.21) / [1 + (71.30/105.21^2)] = 0.421.$$

Empirical Bayes Method

STEP 8: Estimate $Var\{\delta\}$ and $Var\{\theta\}$.

$$Var(\delta) = 71.30 + 44 = 115.30.$$

$$Var(\theta) = 0.421^2\,[(44/44^2) + (71.30/105.21^2)]\,/\,[1 + (71.30/105.21^2)]^2 = 0.0050.$$

Empirical Bayes Method

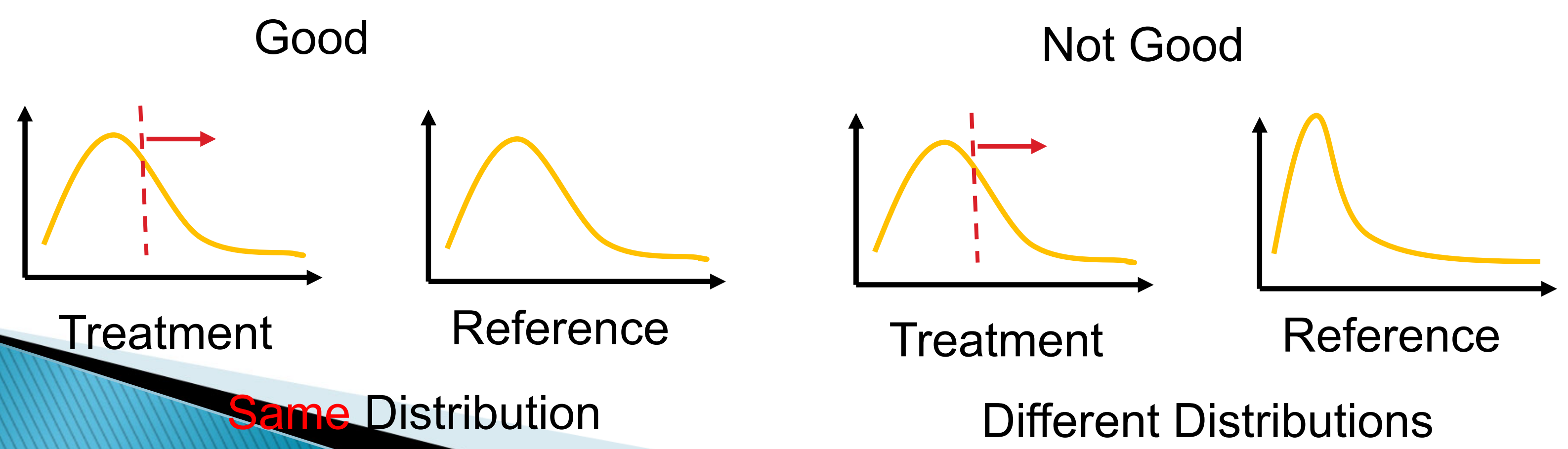
TABLE 8 Estimates of Safety Effect for Groups of Conversions

Group Characteristic Before Conversion/Jurisdiction	Count of Crashes During Period After Conversion		Crashes Expected During After Period Without Conversion (Standard Deviation)		Index of Effectiveness (Standard Deviation)		Percent Reduction in Crashes	
	All	Injury	All	Injury	All	Injury	All	Injury
Single Lane, Urban, Stop Controlled								
Bradenton Beach, FL	1	0	9.9 (3.6)	0 (0)				
Fort Walton Beach, FL	4	0	16.9 (3.9)	2.7 (1.1)				
Gorham, ME	4	0	6.8 (1.4)	0.9 (0.4)				
Hilton Head, SC	9	0	42.8 (6.0)	8.2 (1.9)				
Manchester, VT	1	1	1.7 (0.7)	0 (0)				
Manhattan, KS	0	0	4.2 (1.2)	1.2 (0.5)				
Montpelier, VT	1	1	4.3 (1.8)	1.1 (0.6)				
West Boca Raton, FL	7	0	8.1 (3.0)	2.6 (1.3)				
Entire group (8)	27	2	94.6 (9.0)	16.6 (2.6)	0.28 (0.06)	0.12 (0.08)	72	88
Single Lane, Rural, Stop Controlled								
Anne Arundel County, MD	14	2	24.6 (4.0)	6.2 (1.7)				
Carroll County, MD	4	1	15.2 (2.6)	3.2 (0.9)				
Cecil County, MD	10	1	14.3 (2.9)	5.6 (1.4)				
Howard County, MD	14	1	36.7 (5.5)	7.7 (2.1)				
Washington County, MD	2	0	14.4 (3.1)	4.2 (1.3)				
Entire group (5)	44	5	105.2 (8.4)	26.9 (3.4)	0.42 (0.07)	0.18 (0.09)	58	82
Multilane, Urban, Stop Controlled								
Avon, CO	3	0	19.9 (4.9)	0 (0)				
Avon, CO	17	1	12.2 (3.1)	0 (0)				
Vail, CO	14	—	19.1 (4.4)	—				
Vail, CO	61	—	50.9 (7.6)	—				
Vail, CO	8	—	9.8 (2.1)	—				
Vail, CO	15	—	11.8 (2.3)	—				
Entire group (6)	118		123.7 (11.0)	n/a	0.95 (0.10)	n/a	5	n/a
Urban, Signalized								
Avon, CO	44	1	49.8 (7.0)	5.4 (1.7)				
Avon, CO	13	0	30.1 (5.7)	2.3 (1.0)				
Avon, CO	18	0	52.1 (7.0)	5.3 (1.7)				
Gainesville, FL	11	3	4.8 (1.5)	1.3 (0.5)				
Entire group (4)	86	4	131.7 (10.9)	15.0 (2.7)	0.65 (0.09)	0.26 (0.14)	35	74
All conversions (23)	275	12	454.6 (19.8)	58.5 (5.1)	0.60 (0.04)	0.20 (0.06)	40	80

Final Overall Results

Empirical Bayes Method

- **Caution**
  - The EB method will be biased if the characteristics between the treatment and reference groups are very different (i.e., sample mean, dispersion and distribution of the observed populations – see below)
  - In practice, if an observation meets one or more treatment criteria, it will not be included in the reference group. Thus, this means that the characteristics will most likely be different.



Full Bayes Method

With the advancements in computing power and the application of the Markov Chain Monte Carlo (MCMC) simulation, developing Full Bayes (FB) models is now very easy to perform.

The main advantage of using the Bayes method is that the treatment and control groups can be combined into one dataset for the before and after periods, and the effect of the treatment estimated accordingly.

Furthermore, the EB method assumes that the covariate effect on crashes is known with certainty, whereas the Bayes method assumes that the covariates are represented by a distribution (the posterior values to be exact).

Full Bayes Method

With the full Bayes method, the analyst needs to develop a crash-frequency model where the coefficients are estimated using the Bayes estimation method. With this method, all the data, those from before and after periods as well as those from the treatment and reference/control groups are used together. The overall functional form is as presented below:

$$u_{it} = \exp(\mathbf{x}_{it}\boldsymbol{\beta}_{it} + \varepsilon_i)$$

where μ_{it} is the mean of site i and time t; $\mathbf{x}_{it}$ is a vector of covariates for site i and time t; $\boldsymbol{\beta}_{it}$ is a vector of covariates for site i and time t; and, $\exp(\varepsilon_i)$ is the error that can follow a gamma or lognormal distribution.

Full Bayes Method

$$\mu_{it} = \exp\begin{pmatrix} \beta_0 + \beta_1 \ln AADT_{it} + \beta_2 T_i + \beta_3 t + \beta_4 (t - t_{0i}) \mathbf{I}[t > t_{0i}] \\ + \beta_5 T_i t + \beta_6 T_i (t - t_{0i}) \mathbf{I}[t > t_{0i}] + \beta_7 x_{7i} + \ldots + \beta_k x_{ki} \end{pmatrix}$$

Where $T_i = 1$ if the ith is a treatment site and zero otherwise; t is the tth in the study period; t_{0i} is the year in which the countermeasure or treatment was installed (for a site in a control group, this is defined as the same year as that for the treatment group); and, $\mathbf{I}[t > t_{0i}] = 1$ if t belongs to the after period or zero otherwise.

Full Bayes Method

The previous equation can be re-arranged by separating it between the before and after time periods and treatment and control groups:

Control group

$$\mu_{it,\text{Control},B} = \exp\left(\beta_0 + \beta_1 \ln AADT_{it} + \beta_3 t + \beta_7 x_{7i} + \ldots + \beta_k x_{ki}\right)$$

$$\mu_{it,\text{Control},A} = \exp\left(\left(\beta_0 - \beta_4 t_{0i}\right) + \beta_1 \ln AADT_{it} + \left(\beta_3 + \beta_4\right) t + \beta_7 x_{7i} + \ldots + \beta_k x_{ki}\right)$$

Treatment group

$$\mu_{it,\text{treatment},B} = \exp\left(\left(\beta_0 + \beta_2\right) + \beta_1 \ln AADT_{it} + \left(\beta_3 + \beta_5\right) t + \beta_7 x_{7i} + \ldots + \beta_k x_{ki}\right)$$

$$\mu_{it,\text{treatment},A} = \exp\begin{pmatrix} \left\{\beta_0 + \beta_2 - \left(\beta_4 + \beta_6\right) t_{0i}\right\} + \beta_1 \ln AADT_{it} + \left(\beta_3 + \beta_4 + \beta_5 + \beta_6\right) t \\ + \beta_7 x_{7i} + \ldots + \beta_k x_{ki} \end{pmatrix}$$

Full Bayes Method

Then, sum the estimated crashes for the before and after time periods and treatment and control groups:

$$\mu_{TB} = \sum_{it} \mu_{it,\text{treatment},B} \qquad \mu_{TA} = \sum_{it} \mu_{it,\text{treatment},A}$$

$$\mu_{CB} = \sum_{it} \mu_{it,\text{Control},B} \qquad \mu_{CA} = \sum_{it} \mu_{it,\text{Control},A}$$

Calculate the effects using the following 5-step process:

Step 1—calculate R_c

$$R_c = \frac{\mu_{CA}}{\mu_{CB}}$$

Step 2—predict π

$$\pi = \mu_{TB} \times R_c$$

Step 3—estimate θ

$$\theta = \frac{\mu_{TA}}{\pi}$$

Step 4—estimate δ

$$\delta = \pi - \mu_{TA}$$

Step 5—determine the significance of θ and δ

Estimate the 2.5-, 5-, and 10-percentile from the posterior distribution of the index and the difference. Then, compare the values with the nominal condition if the expected reduction (or increase) is statistically significant.

Sample Size

Rule of Thumb

- Make use of the basic principle of inferential statistics that of the normal distribution

$$P\left(\hat{\theta} - 1 \cdot \sigma(\hat{\theta}) \leq \theta \leq \hat{\theta} + 1 \cdot \sigma(\hat{\theta}) \right) \approx 65\%$$
$$P\left(\hat{\theta} - 2 \cdot \sigma(\hat{\theta}) \leq \theta \leq \hat{\theta} + 2 \cdot \sigma(\hat{\theta}) \right) \approx 95\%$$
$$P\left(\hat{\theta} - 3 \cdot \sigma(\hat{\theta}) \leq \theta \leq \hat{\theta} + 3 \cdot \sigma(\hat{\theta}) \right) \approx 99.9\%$$

Four Factors that Need to be Considered

- Variance of the variable being studied
- Size of the effect of interest
- Level of significance (related to type I error)
- Power of a test (related to type II error)

Sample Size

Variance

- Its square root is either standard deviation or standard error
- Standard Deviation: the measure of how variable individual observations are in a sample
- Standard Error: the measure of how variable the mean or proportion is from one sample to another

$$SE = \frac{SD}{\sqrt{N}}$$

Size of Effect

- The expected size of an effect should be assumed
- This is usually based on the results of previous or pilot studies
- Example
  - A treatment is thought to reduce the expected number of crashes by 10% (i.e., $\theta = 0.9$)

Sample Size

Significance Level

- The significance level tells us how likely it is that an observed difference is due to chance when the true difference is 0.

H_0: $\theta_1 = \theta_2$ (no difference)

H_A: $\theta_1 - \theta_2 > 0$

	Do not reject H_0	**Reject H_0**
H_0 is True	Correct Decision 1-α: Confidence level	Type I error α: Significance level
H_0 is False	Type II error β	Correct Decision 1-β: Power of a test

- Sample size can be determined by considering the significance level only.
- However, in order to detect the specific effect of a treatment, the sample size can be determined by considering both significance level and power.

Sample Size

Power of a Test

- Power is the probability that it will correctly lead to the rejection of a false null hypothesis.
- We can think of power as the probability of detecting a true effect.
- Two different aspects of power analysis. One is to calculate the necessary sample size for a specified power. The other aspect is to calculate the power for given a specific sample size.
- Generally, a test with a power greater than 0.8 (or $\beta <= 0.2$) is considered statistically powerful.

Sample Size Calculations

General Approach

First, let us define x_b and x_a as the number of crashes in the before and after periods, and t_b and t_a as the before and after time periods (say in years). Then, let $\mu_b = x_b/t_b$ and $\mu_a = x_a/t_a$ be defined as the number of crashes per unit of time (i.e., Poisson mean over time). Using the methodology proposed by Hauer (2008), one can calculate or examine the sample size based on this relationship $d > 0$, where $d = \mu_b - \mu_a$.

$$\frac{d}{SE(d)} = \frac{\mu_b - \mu_a}{\sqrt{(x_b/t_b^2 + x_a/t_a^2)}} = Z_{\alpha/2}$$

Significance level only

$$\frac{d}{SE(d)} = \frac{\mu_b - \mu_a}{\sqrt{(x_b/t_b^2 + x_a/t_a^2)}} = Z_{\alpha/2} + Z_{\beta}$$

Significance and Power

Sample Size Calculations

$$\frac{d}{SE(d)} = \frac{\mu_b - \mu_a}{\sqrt{(x_b/t_b^2 + x_a/t_a^2)}} = Z_{\alpha/2}$$

Significance level only

$$\frac{d}{SE(d)} = \frac{\mu_b - \mu_a}{\sqrt{(x_b/t_b^2 + x_a/t_a^2)}} = Z_{\alpha/2} + Z_\beta$$

Significance and Power

TABLE 7.3 Combination of significance and power (Kelsey et al., 1986).

Significance (α)	Power ($1-\beta$)	$Z_{\alpha/2} + Z_\beta$
0.01 ($Z_{\alpha/2=0.005} = 2.575$)	0.80	3.417
	0.90	3.857
	0.95	4.221
	0.99	4.902
0.05 ($Z_{0.025} = 1.960$)	0.80	2.802
	0.90	3.241
	0.95	3.605
	0.99	4.286
0.10 ($Z_{0.05} = 1.645$)	0.80	2.802
	0.90	3.241
	0.95	3.605
	0.99	4.286

Example:

On a certain kind of road on which there are 1.5 reported crashes/km-year an intervention is contemplated. The question is how many kilometres of road are needed so that one can be 95% confident that in a before-after study a 10% reduction in expected crash frequency is detected if 3 years of 'before' and 1 year of 'after' data will be used.

Solution:

Let, x_1, x_2 = crash counts for c_1 and c_2 years on n kilometres of road

Subscript 1 and 2 represents 'before' and 'after' period

Then, x_1=1.5*3*n=4.5n

x_2=(1.5)*(0.9)*1*n=1.35n

$$\frac{(x_1/nc_1)-(x_2/nc_2)}{\sqrt{x_1/(nc_1)^2+x_2/(nc_2)^2}}=\frac{(1.5)-(1.35)}{\sqrt{4.5/9n+1.35/n}}\approx 2.0$$

This yields n=330 km.

Therefore, x_1=495 crashes/year and x_2=446 crashes/year are required.

Source:

Hauer, E. (2008) How many accidents are needed to show a difference? Accid Anal Prev 40(4): 1634-5.

Sample Size Calculations for Before-After Studies

Naïve Method

Using a Comparison Group

Empirical Bayes Method

Naïve Method

Two decisions that need to be made

- The number of entities (or accidents) for the treatment group
- The duration of the 'before' and 'after' periods

Precision = Standard error of the estimate, $\sigma(\hat{\theta})$

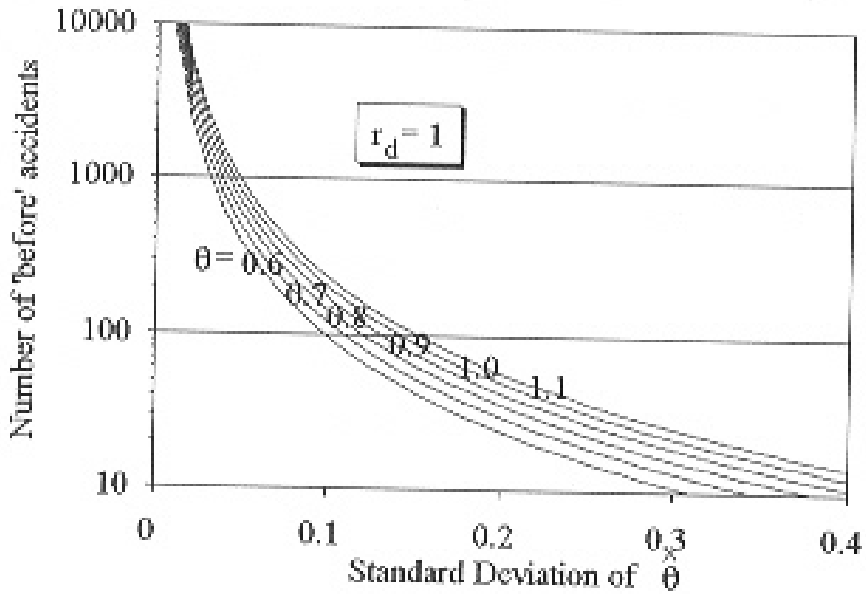
$$\sum \kappa(j) = \frac{\theta / r_d + \theta^2}{\sigma^2(\hat{\theta})} \approx \frac{2}{\sigma^2(\hat{\theta})}$$

$$P\left(\, |\hat{\theta} - \theta| \leq 1 \cdot \sigma(\hat{\theta}) \, \right) = 65\% \qquad P\left(\, |\hat{\theta} - \theta| \leq 2 \cdot \sigma(\hat{\theta}) \, \right) = 95\%$$

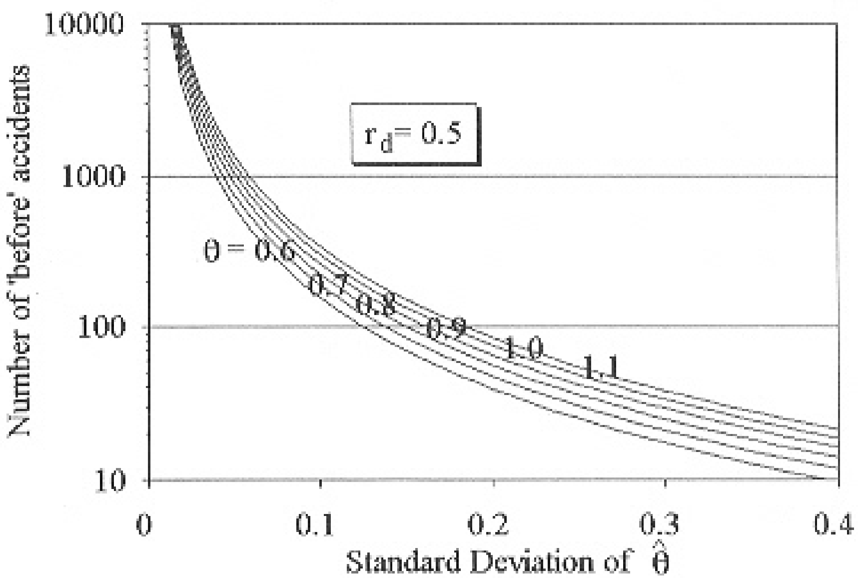
When $\sigma(\hat{\theta}) = 0.1$, we need 200 'before' accidents

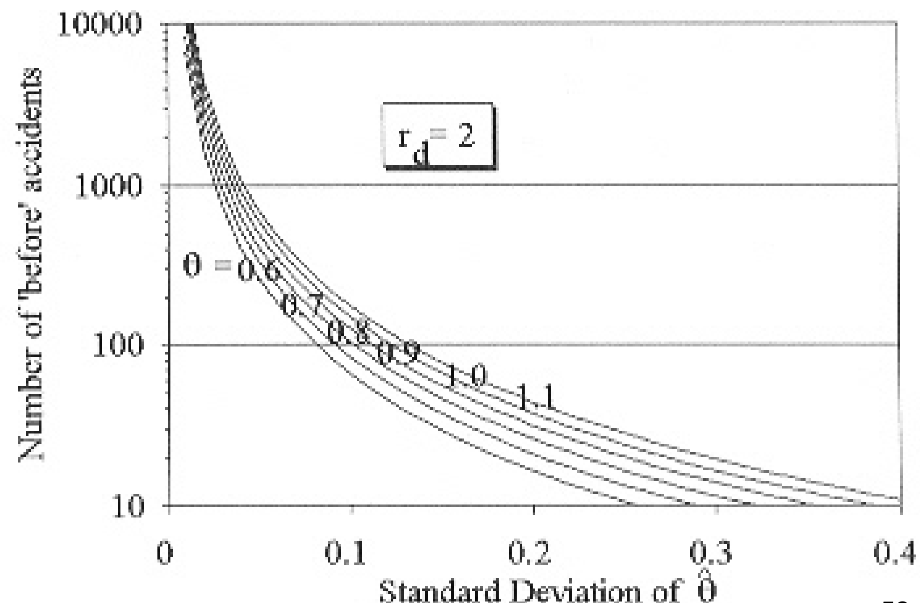
$\sigma(\hat{\theta}) = 0.01$, we need 20,000 'before' accidents

Naïve Method



$$\sum \kappa(j) = fn\left\{ \theta, r_d, \sigma(\hat{\theta}) \right\}$$





Naïve Method

Example: A treatment is thought to reduce the expected number of crashes by 10% (i.e., $\theta = 0.9$). If the before and after period are one year in duration, what is the number of crashes need for the before period for $\sigma(\hat{\theta}) = 0.05$?

$$\sum \kappa(j) = \frac{0.9/1 + 0.9^2}{0.05^2} \approx 700 \text{ crashes}$$

What if the system can provide only 175 accidents per year?
How can we get the same statistical precision $\sigma(\hat{\theta}) = 0.05$?

- Option 1: Increase the 'before' and 'after' periods to 4 years
- Option 2: Increase the 'before' period to 3 years, and the 'after' period to 5.4 years

Are those good options?

Using a Comparison Group

The sample size needed when the study includes a control group, is governed by the terms $\sigma^2\{\hat{\theta}\}$ or $Var\{\theta\}$ and $Var\{\omega\}$

$$\sigma^2\{\hat{\theta}\} = \frac{\theta / r_d + \theta^2}{\sum \kappa(j)} + \theta^2 \left[\frac{1/r_d + 1}{\sum \mu(j)} + \frac{Var(\omega)}{\omega^2} \right]$$

Number of crashes in treatment group

Number of crashes in control group

Variance of odd ratios

odd ratios (usually close to 1)

This is estimated from the control and treatment groups

Using a Comparison Group

Example: Taking the same example as before with $\sigma\{\hat{\theta}\} = 0.05$, now assume the control group contains 5,000 crashes for the before period with $Var(\omega) = 0.001$ and $\omega = 1.0$

The comparison group contributes to the overall variance

$$\theta^2 \left[\frac{1/r_d + 1^2}{\sum \mu(j)} + \frac{Var(\omega)}{\omega^2} \right] = 0.9^2 \left[\frac{2}{5,000} + 0.001 \right] = 0.0011$$

$$\sigma^2\{\hat{\theta}\} = 0.0025 = \frac{\theta / r_d + \theta^2}{\sum \kappa(j)} + 0.0011 = 0.0014$$

$$\frac{\theta / r_d + \theta^2}{\sum \kappa(j)} = 0.0014 \qquad \sum \kappa(j) = \frac{0.9/1 + 0.9^2}{0.0014} = 1,222 \text{ crashes}$$

Empirical Bayes Method

$$\mu_{EB} = w \times \mu + (1 - w) \times y$$

μ_{EB} = Estimate of the expected number of crashes for an entity of interest

μ = Expected number of crashes based on expected on similar entities

y = number of crashes on the entity of interest

w = Weight factor $= \dfrac{1}{1 + \mu / \phi}$

- The sample size issue arises when μ is estimated from a statistical model (a negative binomial model)
- Larger sample size reduces the bias in the dispersion parameter estimate (see next two slides)
- Given the characteristics of crash data, i.e. Low mean and overdispersion, models should be developed with at least 100 observations. Ideally, more than 1,000 observations should be used.

Empirical Bayes Method

TABLE 6.4 Recommended sample size (Lord, 2006).

Population sample mean	Minimum sample size
5.00	200
4.00	250
3.00	335
2.00	500
1.00	1000
0.75	1335
0.50	2000
0.25	4000

NB models estimated using the MLE

Empirical Bayes Method

TABLE 6.5 Recommended minimum sample size for Bayesian Poisson-lognormal models (Miranda-Moreno et al., 2008).

Population sample mean	**Minimum sample size**
≥2.00	20
1.00	100
0.75	500
0.50	1000
0.25	3000

NB/PLN models estimated using the Bayesian method

(Note: if using the FB method, there is no need to use the EB)